Appraisal of ceramic substrates by multivariate tolerance regions

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Abstract. Determination of standards or norms and checking conformance to these standards in terms of both location and spread is a central issue in process control procedures. New observations are tested against such standards in order to determine if they are being met. In this work we apply and expand a methodology based on multivariate tolerance regions to lots of ceramic substrates used in the microelectronics industry. The method provides the appraiser with both graphical and numerical tools which include objective criteria for process control decision making.

Introduction

The case study presented in this article is an analysis of physical dimensions of ceramic substrate plates used in the microelectronics industry. Several measurements are taken on individual substrates in order to decide whether the plates conform to an accepted standard and can be used in the production line. The plates arrive in batches and a decision-making process has to determine the conformance to standards of the individual plates and of the batches.

The determination of standards or norms is a central issue in quality control procedures (e.g. Eisenhart *et al.*, 1947; Leurgans, 1978; Fuchs & Kenett, 1987). The norms can be derived from a process capability study. The resulting sample provides a reference basis to which new observation are compared. Based on the values of all the measured variables it is decided (with prespecified margins of error) whether the new observation originates from a target reference population. The formulation of the process control problem in terms of multiple variables is very common. However, even when it is recognised that there are several dimensions the procedures applied are often univariate on key characteristics. Also common is the application of a series of univariate tests performed either simultaneously or in a stepwise fashion. The loss of information in the first case and the difficulty to assess the probability of error in the second are obvious. Another common method is to use data on all the variables to test the hypothesis of equality of means when in fact the question of interest is whether the tested observation belongs to the target population. An equivalent formulation of the latter question is to define a region covered by the target standard population and to test whether the new observation is contained in that region. Formally this defines a tolerance region for which we can assert with probability δ that it contains at least a proportion P of the population of interest.

The approach in this case study relies on multivariate tolerance regions with special attention devoted to the batch structure of the ceramic substrates. The product and manufacturing steps are described in the next section. Subsequent sections present the process control procedures and their application to our case study. We conclude with a discussion of the results.

Description of the substrates and of the manufacturing flow

Raw materials used in the manufacturing of hybrid microcircuits consist of components, dyes, pastes and ceramic substrates. The substrate plates undergo a process of printing and firing during which layers of conductors, dielectric, resistors and platinum or gold are added to the plates. Subsequent production steps consist of laser trimming, components mounting and reflow soldering or chip enwire bonding. The last manufacturing stage is the packaging and sealing of completed modules.

A set of identical modules are manufactured simultaneously on one substrate plate. The modules are separated before final packaging along lines inscribed on the substrate in a laser inscribing process. Exact positioning and dimensions of these lines determine, to a large extent the success of the printing and components mounting operations. Substrates come in batches of about 100 items. The sampling scheme used is MIL-STD-105D with an AQL of 1% yielding a sample size of 13 and acceptance criterion of 0 under inspection level II and normal inspection. The 13 items sampled are not only tested on a go/no go basis but actual measurements are recorded. Because of the variety of modules manufactured specifications of the substrate dimensions vary from batch to batch in nominal target values but not in specifications width. In this case study we shall use deviations of the actual measurements from the center of the specifications. This normalisation enables us to treat all batches on a uniform basis. The assumption made here is that there are no location effects. An early study of this issue validated this assumption.

Five dimensions are considered here, with labels (a, b, c), (W, L) (see Fig. 1). The first three are determined by the laser inscribing process. The last two are outer physical dimensions. These outer dimensions are measured on a different instrument than the first three, using a different sample of substrates. The two sets of measurements have in common the batch affiliation.

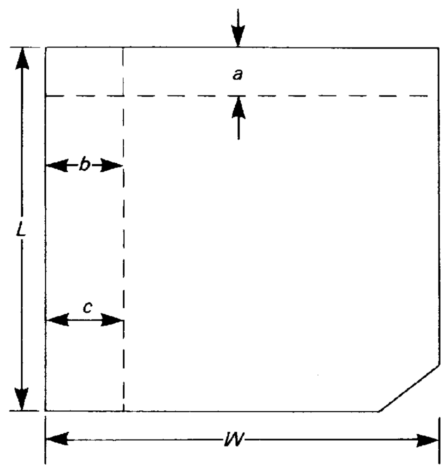

Fig. 1. Ceramic substrate layout.

There are two possible outcomes to the decision process: accept the batch as is or reject the batch and start a 100% screening of substrates. The need for this decision process comes from our lack of confidence in the stability of the substrate manufacturing process. During screening, we have to decide for each item whether it meets our standard or has to be scrapped. In the sampling mode we want to reach a decision (accept or screen) as soon as possible. Therefore a curtailed inspection scheme should be used. The invocation of MIL-STD-105D is due to a contractual requirement. The standard used in our analysis consists of measurements on a prototype batch of substrates which, when used in the subsequent manufacturing stages, proved to be of good quality in that, that later no problems related to physical dimensions surfaced. From this 'reference sample' we determined standard achievable dimensions including the correlation structure between the measured variables.

Multivariate tolerance regions of individual and grouped data

Let $X_1, \ldots, X_n$ be a 'reference sample' of ceramic substrates as defined above. We assume that the sample is composed of n independent and identically distributed observations from a multivariate normal distribution of dimension q with unknown mean μ and covariance matrix Σ. The dimension q is 3 when we study the (a, b, c) measurements and 2 for the (W, L) measurements. Critical values for (P, δ)-tolerance regions can be obtained from John (1962). Tolerance regions have been applied by Fuchs & Kenett (1987) to several multivariate quality control examples with reference samples. In this case study we encounter several specific problems which did not occur in the examples analysed in the latter article. First, the reference sample is relatively small and therefore the adequacy of the large sample approximation presented in Fuchs & Kenett (1987) has to be tested. Secondly, the fact that the data are grouped requires decision making both at the level of individual observations as well as for entire batches. Finally, we address the question of curtailed inspection which can save considerable effort, as illustrated in our case study.

Let X_{ih} be the value of the i^{th} variable for the h^{th} observation and S the empirical covariance matrix with elements s_{ij}, i.e.

$$s_{ij}=\sum_{h=1}^{n}(X_{ih}-\overline{X}_i)(X_{jh}-\overline{X}_j)/(n-1).$$

The means $\overline{X}_i$, $\overline{X}_j$ are the i^{th} and j^{th} elements, respectively, of the sample mean vector $\overline{X}=(\overline{X}_1, \ldots \overline{X}_q)$. It has been shown by John (1962) that if we ignore terms of order two in $(1/n)$ the tolerance region can be defined by the Y-vectors satisfying the inequality

$$(Y-\overline{X})'S^{-1}(Y-\overline{X})\leq k, \tag{1}$$

where k is a function of the prespecified (P, δ) and of the dimensionality of the problem (q). The left hand side of (1) is the Mahalanobis (squared) distance. The critical value k is defined as follows: Let $g(\phi, f, m)$ be the ϕ^{th} percentile of the non-central chi-squared distribution with m degrees of freedom and parameter of noncentrality f. As a special case $g(\phi, 0, m)$ is the ϕ^{th} percentile of the central chi-squared distribution, i.e.

$$\Pr(\chi^2_m\leq g(\phi, 0, m))=\phi \tag{2}$$

The critical value k is given by

$$k=g(P, q/2n, q)(n-1)q/g(1-\delta, 0, (n-1)q) \tag{3}$$

For a given sample size, the tolerance limits k can be computed for each combination of (P, δ)-values. Those limits can be drawn on multivariate control charts of the Mahalanobis distances from the observations to the reference sample. When the sample size is relatively large (say, over 50) and the number of attributes is moderate $(q<7)$ it has been shown in Fuchs & Kenett (1987) that the use of the inverse of the central chi-square distribution in the numerator of (3) yields adequate approximations. Using that approximations the critical value k is given by:

$$k_0 = g(P, 0, q)(n-1)q/g(1-\delta, 0, (n-1)q) \quad (4)$$

which is much easier to calculate. However, in our case, the reference sample size is relatively small ($n=13$) and it is important to study the adequacy of the approximations for such cases as well. Table 1 presents both exact and approximate results for our case (i.e. $n=13$ and $q=3$). We note that since in the second case of our study the parameter of non-centrality is smaller, the performance of the approximation with $n=13$ and $q=2$ will be better than that presented in Table 1.

Table 1. ($n=13$, $q=3$)

P	δ	k	k_0	$P_0:\delta$	$\delta_0:P$
0·60	0·90	4·296	4·136	0·582	0·782
0·60	0·95	4·735	4·559	0·582	0·934
0·70	0·90	5·343	5·146	0·683	0·872
0·70	0·95	5·890	5·672	0·683	0·934
0·90	0·90	9·114	8·778	0·889	0·873
0·90	0·95	10·045	9·675	0·889	0·934
0·95	0·90	11·395	10·974	0·943	0·872
0·95	0·95	12·560	12·096	0·943	0·934

The critical values (exact and approximated) are in columns k and k_0, respectively. The effect of the approximation on the two probabilities are illustrated in the last two columns. Column $P_0:\delta$ contains the actual proportion of coverage obtained by the use of k_0 with δ fixed at its postulated value. For example, when $P=0{\cdot}90$ and $\delta=0{\cdot}95$ the exact and approximate critical values are 10·045 and 9·675 respectively. The use of $k_0=9{\cdot}675$ corresponds to a region for which we can assert with probability 0·95 that the coverage is 0·889 (instead of the postulated 0·90 attained with the exact $k=10{\cdot}045$). Similarly from column $\delta_0:P$ we read that for the same k_0 the actual confidence level is 0·934 (instead of the postulated 0·95). We observe that the differences between P and $P_0:\delta$ vary from 0·018 (for $P=0{\cdot}6$) to 0·007 (for $P=0{\cdot}95$). Similarly the differences between δ and $\delta_0:P$ vary from 0.034 (for $\delta=0{\cdot}8$) to 0·014 (for $\delta=0{\cdot}95$).

Similar computations are presented in Fuchs & Kenett (1987) for $q=6$ and $n=20$, 50, 100, 500. It is interesting to observe that the performance of the approximations for the small sample size of $n=13$ with $q=3$ is slightly better than that achieved with $n=20$ and $q=6$. This is not surprising if we notice that the parameter of non-centrality $q/2n$ is larger in the case $n=20$, $q=6$. We can thus conclude that even when the reference sample is small, the use of the approximation is appropriate in the construction of multivariate tolerance regions in general and in quality control in particular.

The rejection strategy defined by (1)–(4) above refers to individual observations, i.e. an observation Y which satisfies (1) is not rejected. Jackson (1985), following Hotelling (in, Eisenhart *et al.*, 1947), deals with grouped data and presents the multivariate analog of supplementation of the standard chart for average with a companion chart for ranges or standard deviations. The two measured are achieved in the multivariate case by the decomposition of the overall variability computed from the tested group (T^2) into two measures of variability M^2 and D^2 such that $T^2=M^2+D^2$. M^2 is a measure of the distance between the mean of the group of tested observations and the mean of the reference sample, while D^2 is a generalised measure of dispersion of the group around its own mean.

For a group of m observations Y_i, $i=1, \ldots, m$ with mean $\bar{Y}$ we have

$$T^2=\sum_{i=1}^{m}(Y_i-\bar{X})'S^{-1}(Y_i-\bar{X})$$

$$D^2=\sum_{i=1}^{m}(Y_i-\bar{Y})'S^{-1}(Y_i-\bar{Y})$$

$$M^2=m(\bar{Y}-\bar{X})'S^{-1}(\bar{Y}-\bar{X})$$

The construction of T^2 and its decomposition were presented by Jackson (1985) and Hotelling (1947) with reference to hypothesis testing and not in the context of tolerance regions. Jackson (1985) mentions that T^2, M^2 and D^2 have asymptotic χ^2 distribution with mq, q, and $(m-1)q$ degrees of freedom, respectively. The use of the asymptotic χ^2-distribution is equivalent to stating that the reference sample is large enough such that its covariance matrix (S) closely approximates Σ. This approximation will not hold in general for $n=13$. Furthermore, as shown in John (1962), Fuchs & Kenett (1987) and in equations (2)–(4) above, more accurate critical values are not difficult to obtain.

In addition, the decomposition of T^2 can be perceived as a descriptive tool rather than as an exact testing hypotheses procedure. We propose the use of the two ratios:

$$I_1=M^2/T^2$$

and

$$I_2=D^2/T^2$$

as indices for detecting whether the overall variability is mainly due to the distances between means of the tested group and those of the reference sample, or to variability within the tested group. When no source of variability supersedes significantly the other, the two indices should be approximately of the same order of magnitude.

The direct application of the multivariate quality control methods presented above to grouped data requires the inspection of the entire group. However, a complete inspection may be wasteful when inspection of the first items suffices to reject the tested batch. A stopping rule for curtailed inspection is presented below.

Let $Y_i, \ldots, Y_m$ be the tested group. The curtailed inspection technique is based on the inverse of the negative binomial distribution. The m observations are tested sequentially. Define $Z_i=1$ if the Mahalanobis distance of the i-th tested observation is greater than k and $Z_i=0$ otherwise. We assume that the observations are independent. The critical value k defines a region which covers a proportion P of the standard population (with probability δ). Let us define the null hypothesis to state that:

$$\Pr(Z_i=0)\geq P.$$

Let α be a prespecified significance level for testing the null hypothesis. For each $j\leq m$, let $R(j)$ be the first observation such that:

$$\Pr\left(\sum_{i=1}^{R(j)} Z_i\leq j\right)\geq\alpha, \tag{5}$$

namely, if at the $R(j)$-th observation we observed the j-th value of $Z=1$ we curtail the inspection and declare that the proportion of coverage is less than P. The probability in (5) defines the negative binomial distribution and $R(j)$ is computed from its inverse. Although a table of $R(j)$ can be computed for each $1\leq j\leq m$, in practice we need only compute the function for $j\leq j_0\leq m$, where j_0 is the minimal value for which

$$\Pr\left(\sum_{i=1}^{m} Z_i \leq j_0\right) \geq \alpha, \tag{6}$$

The value j_0 defines the minimal number of individual rejections (out of m) for which we reject the null hypothesis. We would thus certainly reject the null hypothesis if we observe more rejections and/or if the number of inspected items up to the j_0 rejection is less than m. Table 2 presents the $R(j)$ values for $m=13$, $P=0{\cdot}95$ and $\alpha=0{\cdot}05$, $0{\cdot}01$, and $0{\cdot}001$.

Table 2.

	$\alpha=0{\cdot}05$	$\alpha=0{\cdot}01$	$\alpha=0{\cdot}001$
$j=2$	7	3	—
$j=3$	13	9	4
$j=4$	—	13	9
$j=5$	—	—	13

We observe that for the three values of α, $j_0=3$, 4 and 5, respectively. If for example we adopt $\alpha=0{\cdot}01$, we reject the null hypothesis at or before the third observation if $\Sigma Z_i=2$ up to that stage. We reject H_0 at or before the 9th observation if $\Sigma Z_i=3$ (provided that we did not reject it already from the results of the first two or three observations). Finally, whenever we obtain the fourth value of $Z=1$, we curtail the inspection.

Results in the case study

Data on dimensions of several lots of ceramic substrates will be now analysed using the procedures presented in the previous section. Robustness to normality assumptions is discussed in Fuchs & Kenett (1987) and references therein. The substrates are produced in lots of varying sizes. The first production lot of 13 units (labelled REFERENCE) proved to be of extremely good quality yielding an overall smooth production with no scrap and repairs. This first lot was therefore considered a 'standard' to be met by all following lots. Any evidence of lots not meeting this standard should give a signal for action.

The means (with respect to the center of the nominal specifications) and the S^{-1} matrix of the REFERENCE sample are as shown in Table 3.

Table 3. Reference measurements

	a	b	c
Means (X)	−1·000	0·615	0·923
S^{-1}-matrix	1·081	0·100	−0·432
	0·100	1·336	0·063
	−0·432	0·063	2·620
	W	L	
Means (X)	0·000	0·000	
S^{-1}-matrix	0·349	−0·095	
	−0·095	0·571	

In the first stage of the analysis distances from REFERENCE are to be computed for individual observations. The computation of the Mahalanobis distance is illustrated with the (*a*, *b*, *c*) measurements of first observation of SAMPLE 3. The vector of measurements was (200, 551, 552) and since the center of the specifications was (200, 550, 550) the *Y*-vector was (0·000, 1·000, 2·000) yielding

$$(Y-\bar{X})'=(1{\cdot}000,\ 0{\cdot}385,\ 1{\cdot}077)$$

and

$$(Y-\bar{X})'S^{-1}(Y-\bar{X})=3{\cdot}515.$$

We now present graphically (in Figs 2–5) the results obtained in the analysis of the (*a*, *b*, *c*) measurements in three subsequent batches of sizes 4, 9, 13 (SAMPLE 30, SAMPLE 7, SAMPLE 12, respectively). Those particular samples were selected since the first two of them illustrate some of the problems which the investigator has to be prepared to encounter in the analysis of field data. SAMPLE 30 has only four observations on (*a*, *b*, *c*) but 13 observations on (*W*, *L*). The inspection on (*a*, *b*, *c*) has been curtailed after the fourth observation. The Mahalanobis distances for those four observations were 4·56, 3282·32, 8·47 and 6618·26 respectively. Next, SAMPLE 7 had in fact 13 observations but only 9 are presented here. The reason is that the last four very large distances from the REFERENCE result quite obviously from errors in recording. More specifically, the first 9 observations are around (200, 550, 550) with a maximum deviation of ±6, while the last four observations are around (250, 550, 550). SAMPLE 12 is a regular sample of 13 observations with no special characteristics.

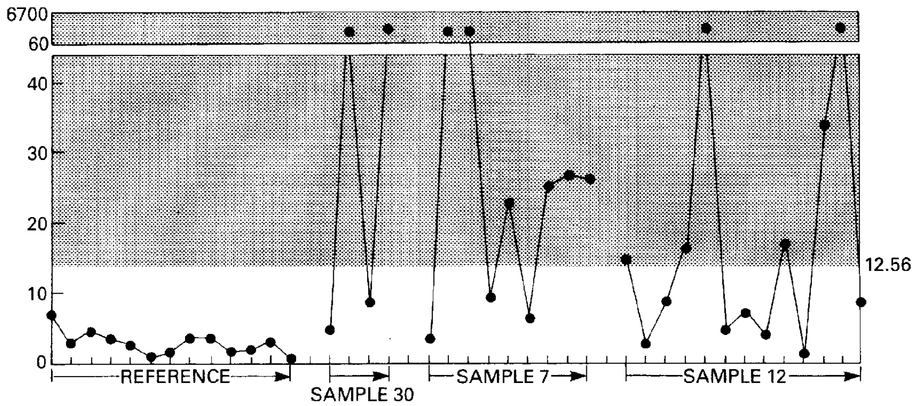

Fig. 2. Multivariate tolerance intervals for (*a*, *b*, *c*) measurements ($\delta=0{\cdot}95$, $P=0{\cdot}95$).

Fig. 2 presents the Mahalanobis distances computed for each of the 39 units. The tolerance limit *k*, corresponding to $\delta=0{\cdot}95$ and $P=0{\cdot}95$, is indicated on the plot. We observe that unlike the units of REFERENCE, the other 26 observations have several distances exceeding the critical values. The data for each of the attributes separately (*a*, *b*, *c*) are presented in Figs 3–5, respectively, with tolerance limits indicated as in the multivariate case ($\delta=0{\cdot}95$, $P=0{\cdot}95$). (For the computation of univariate tolerance intervals see, e.g. Eisenhart *et al.*, 1947; Hald, 1962). In order to produce a figure on a reasonable scale outlying values were isolated in separate 'boxes'. In practice such extreme values should trigger an investigation on the specific units.

A cross check of Fig. 2 with Figs 3–5 indicates how the overall picture presented by the Mahalanobis distances is affected by the one dimensional values and their

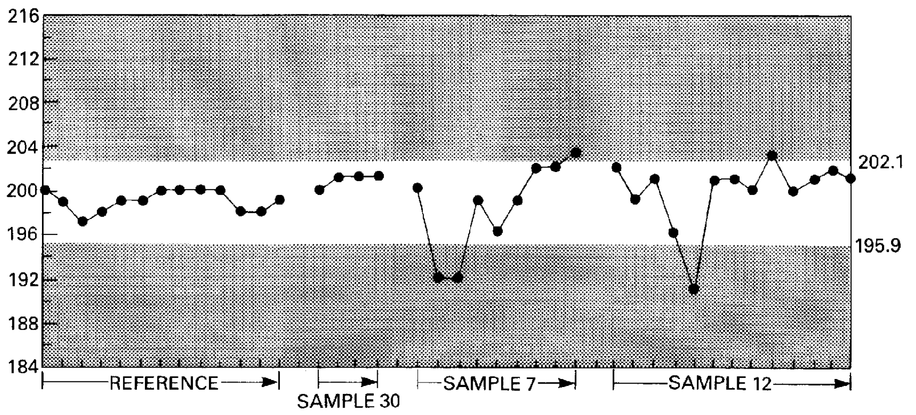

Fig. 3. Tolerance intervals for the *a*-measurements (δ=0·95, *P*=0·95).

correlations. For example the first observation of SAMPLE 12 is on all three dimensions within the 95% tolerance limits of the reference population. However, its multivariate distance is beyond the 95% critical limit. On the other hand the fourth observation of SAMPLE 7 lies within the 95% tolerance region although measure (*b*) exceeds the univariate critical values.

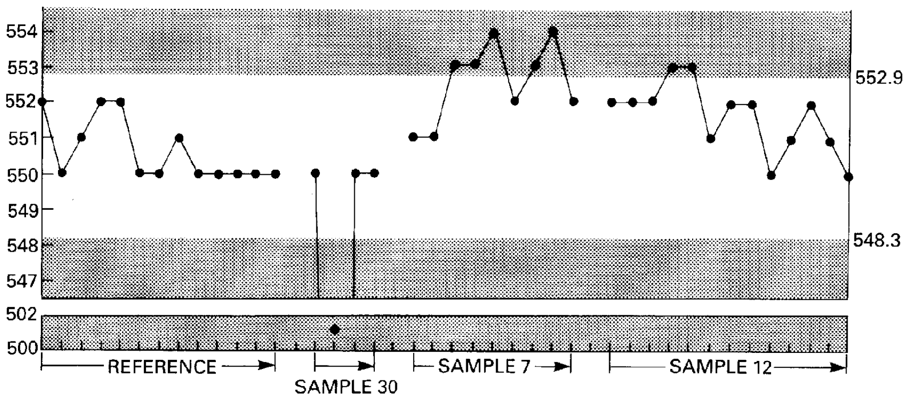

Fig. 4. Tolerance intervals for the *b*-measurements (δ=0·95, *P*=0·95).

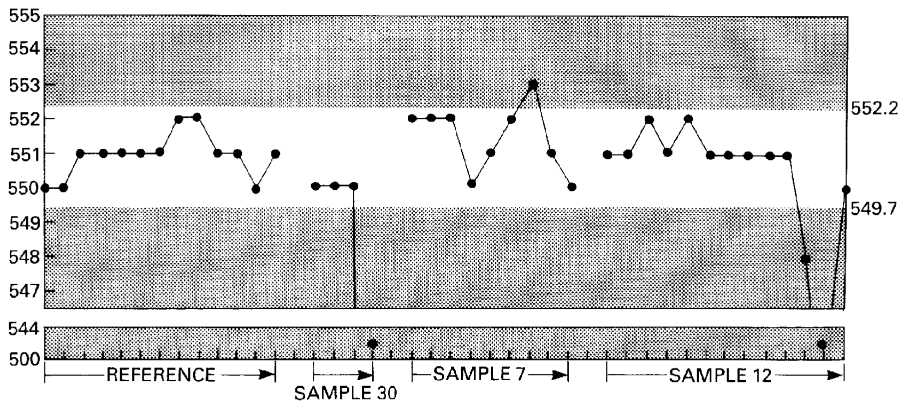

Fig. 5. Tolerance intervals for the *c*-measurements (δ=0·95, *P*=0·95).

The results of the analyses performed on the 30 groups of ceramic substrate measurements are summarised in Tables 4–5. Based on the results on small sample approximations for k, the Mahalanobis distances are compared to the computed k_0-values. The tables also include information on the number of items to be inspected if we adopted a curtailed inspection technique with α=0·01 and a postulated P=0·95. The last columns of the tables present the decompositions of the T^2-values and the related indices I_1 and I_2.

Table 4. Summary results for the (a, b, c) measurements. REJ=Number of rejections using k_0 and (P=0·95, δ=0·90). C.I=Items inspected using curtailed inspection, α=0·01

SMPL	m	Means	REJ	C.I	T^2	M^2	D^2	I_1	I_2
REF	13	(−1·0, 0·6, 0·9)	0	13	36·0	0·0	36·0	0·00	1·00
1	13	(−2·7, 0·4, 0·4)	3	2	468·2	172·0	296·2	0·37	0·63
2	13	(2·5, −0·4, 0·2)	10	2	326·6	235·8	90·8	0·73	0·27
3	13	(3·7, 1·7, 0·5)	10	2	489·3	373·2	116·1	0·76	0·24
4	13	(0·0, −0·2, −0·1)	1	13	89·2	68·8	20·4	0·77	0·23
5	13	(−0·1, 0·2, 0·3)	0	13	59·3	34·3	25·0	0·57	0·43
6	16	(−2·7, 1·8, 2·0)	13	2	438·1	154·0	284·1	0·35	0·65
7	13	(14·7, 2·2, 1·6)	10	3	12045·1	3466·2	8578·9	0·29	0·71
8	13	(0·4, −3·2, −4·3)	13	2	3784·4	1315·2	2469·6	0·35	0·65
9	14	(0·5, 0·4, 1·1)	0	14	68·1	32·2	35·9	0·47	0·53
10	12	(0·7, 1·3, 0·3)	2	12	177·4	72·0	105·4	0·41	0·59
11	13	(1·2, 2·2, 1·0)	6	2	133·8	113·5	20·3	0·84	0·16
12	12	(−0·2, 1·6, 0·6)	6	5	345·6	51·4	294·3	0·15	0·85
13	13	(−0·7, −4·8, 1·3)	13	2	687·3	500·8	186·5	0·73	0·27
14	13	(0·3, 1·4, 0·7)	5	8	135·8	41·9	93·8	0·31	0·69
16	13	(0·7, 3·0, 1·8)	10	2	197·3	164·7	32·6	0·83	0·17
17	13	(0·7, 1·5, 0·9)	3	8	97·9	59·1	38·7	0·60	0·40
18	13	(0·5, 1·1, 0·2)	1	13	77·4	66·6	10·8	0·86	0·14
19	13	(1·7, −0·6, −0·1)	10	13	277·1	185·6	91·4	0·67	0·33
20	13	(0·0, 0·3, 0·0)	0	13	61·1	54·7	6·3	0·90	0·10
21	14	(0·6, 1·6, 1·6)	0	14	89·4	62·7	26·6	0·70	0·30
22	6	(−0·3, 0·2, 0·3)	0	6	20·5	11·8	8·7	0·58	0·42
23	5	(0·0, 0·0, 0·0)	0	5	22·8	22·8	0·0	1·00	0·00
24	4	(−0·5, 0·0, 0·0)	0	4	16·9	9·2	7·7	0·54	0·46
25	13	(0·0, 0·7, 0·7)	0	13	29·8	18·7	11·1	0·63	0·37
26	4	(0·3, 0·5, 0·5)	0	4	17·2	10·4	6·7	0·61	0·39
27	6	(0·0, 0·8, 0·8)	0	6	15·5	7·7	7·8	0·50	0·50
29	13	(0·2, −0·2, 0·0)	2	13	89·0	72·0	17·0	0·81	0·19
30	4	(0·3, −12·3, −12·3)	2	4	9913·6	2861·9	7071·7	0·29	0·71

We note from the results presented in the tables that in most cases the non-standard batches could have been detected in very early stages of the inspection process. In several of the groups in which there are considerable number of rejections, the first two inspected items yielded Mahalanobis distances larger than k_0—and the inspection could thus have been curtailed.

We mention in this context that some of the batches deviate considerably from the reference samples. On the other hand, the 13 (W, L)—observations in SAMPLE 18 and the 5 (a, b, c)-observations in SAMPLE 23 were at their respective centres of specifications. The T^2—values in the two cases differ however, since in the reference samples the means of the (W, L)-measurements were at their centres of specification while the means of the (a, b, c)—measurements deviate from the centres of specification.

The indices I_1 and I_2 are helpful in identifying sources of variability. We illustrate the assertion with two examples based on the (W, L) measurements. In SAMPLE 4 we conclude from the very low value of I_2 (0·02) that there is almost no variability within

Table 5. Summary results for the (W, L)-measurements. REJ=Number of rejections using k_0 and $(P=0{\cdot}95, \delta=0{\cdot}90)$. C.I=Items inspected using curtailed inspection, $a=0{\cdot}01$

SMPL	m	Means	REJ	C.I	T^2	M^2	D^2	I_1	I_2
REF	13	(0·0, 0·0)	0	13	24·0	0·0	24·0	0·00	1·00
1	13	(13·5, 4·5)	13	2	1082·1	821·9	260·2	0·76	0·24
2	13	(13·8, 0·6)	13	2	1113·0	842·5	270·5	0·76	0·24
3	13	(16·1, 4·5)	13	2	1300·9	1156·1	144·8	0·89	0·11
4	13	(4·4, 4·4)	12	2	187·0	182·5	4·5	0·98	0·02
5	13	(5·5, 4·8)	11	5	276·3	239·8	36·5	0·87	0·13
6	14	(16·8, 7·5)	14	2	2666·3	1491·8	1174·5	0·56	0·44
7	13	(10·3, 0·3)	12	3	1112·2	475·2	637·0	0·43	0·57
8	13	(13·5, 0·7)	13	2	1325·3	803·1	522·2	0·61	0·39
9	13	(3·2, 3·2)	6	3	113·6	96·1	17·5	0·85	0·15
10	8	(0·6, 0·6)	0	8	4·5	1·1	3·4	0·24	0·76
11	5	(0·4, 1·2)	0	5	12·8	3·9	8·9	0·31	0·69
12	13	(11·3, −6·4)	13	2	1529·3	1062·1	467·2	0·69	0·31
13	13	(−4·0, 15·8)	13	2	2438·7	2095·8	343·9	0·86	0·14
14	13	(9·2, −5·6)	12	2	1079·4	749·4	330·0	0·69	0·31
15	13	(4·1, 3·7)	8	2	181·7	141·6	40·1	0·78	0·22
16	13	(0·7, 0·0)	0	13	10·1	2·2	7·9	0·22	0·78
17	13	(9·1, −1·5)	12	3	498·6	422·8	75·8	0·85	0·15
18	13	(0·0, 0·0)	0	13	0·0	0·0	0·0	—	—
19	13	(11·1, 3·1)	11	2	1426·6	550·2	876·4	0·39	0·61
20	13	(−0·3, −0·7)	0	13	43·1	3·5	39·6	0·08	0·92
21	13	(0·1, 1·2)	1	13	31·4	11·1	20·3	0·35	0·65
22	13	(−2·0, −2·6)	0	13	62·1	56·0	6·1	0·90	0·10
23	13	(1·1, 1·2)	0	13	17·4	12·6	4·8	0·72	0·28
24	13	(−0·2, 0·5)	0	13	13·7	2·7	11·8	0·20	0·80
25	13	(0·9, 0·8)	0	13	14·5	7·3	6·8	0·53	0·47
26	13	(1·1, 0·8)	0	13	13·8	8·3	5·5	0·60	0·39
27	6	(1·5, 1·3)	0	6	10·4	8·5	1·9	0·82	0·18
28	13	(−35·9, −43·9)	12	2	38288·2	15025·8	23262·4	0·39	0·61
29	13	(−0·2, −0·3)	0	13	9·0	1·1	7·9	0·12	0·88
30	13	(−0·1, −1·1)	0	13	25·2	8·4	16·8	0·33	0·67

the sample. The large number of rejections (12 out of 13) indicate an almost constant bias of the magnitude of the sample means with respect to the reference sample (and, in this case, also with respect to the centers of specification). On the other hand, in SAMPLE 20, the variability is almost entirely due to the variability within the sample with the (W, L)-means being very close to those in the reference sample.

The variability of the (W, L)-measurement in SAMPLE 20 did not suffice to yield rejections of the individual substrates. In other cases, such as in the (a, b, c)-measurements in SAMPLE 12, the deviations were large enough to generate six rejections. Further analyses in a similar manner can be performed on the indices of the other samples.

We note that several measurements differed so drastically from the standard, that errors in recording is to be suspected. In the presentation of Figs 2–5 we mentioned the last four (a, b, c)-measurements of SAMPLE 7. Not surprsingly, the resulting T^2 is very large. We suspect errors in recording also in two observations of SAMPLE 30, but in this case the errors, if at all, occurred at specific measurements (the b-measurement of the second observation and the c-measurement of the fourth). The 13 (W, L) observations in SAMPLE 28 yielded a very large T^2-value due to large deviations from the standard. The substrates were found to be smooth and not inscribed and thus no (a, b, c)-measurements are available. The batch has obviously been rejected. Data on the (a, b, c)-measurements were also not available for SAMPLE 15.

Discussion

In most statistical process control applications the problems are inherently multivariate. However, multivariate analyses are not routine in quality control and, as stated in Jackson (1985), there is a need for more practical tools to handle the multidimensional case. This case study is an attempt at tackling the multivariate process control problem using explicitly the correlation structure in the data. A treatment of specifications setting in terms of multiple attributes was presented in Kenett & Halevy (1984).

The approach taken here leads to the more fundamental question about the appropriate statistical inference procedure to be used. The controversy on this issue is still unsettled (Johnstone, 1987). We believe that tolerance regions provide a practical tool building on the population concept.

The ceramic substrates data analysed here consist of a basic reference sample derived from a prototype batch produced by engineering under a tight laboratory environment. Subsequent samples were drawn from batches produced by manufacturing personnel using procedures set by engineering. The analysis presented here shows problems associated with the initial production stages where assignable causes not anticipated by engineering emerge. The multivariate tools used in this article helped smooth up this transition phase by providing a real time picture of discrepancies between ongoing production and what was achieved in the development laboratory.

In this case study we applied multivariate tolerance regions using central chi-squared approximation that is both precise and practical. The batch structure of the data led us to develop a simple curtailed sampling scheme that supplements the tolerance region concept in a general setting and provides an efficient decision rule. Practitioners interested in applying these techniques will find that electronic spreadsheets can make these tools readily available to the workers on the production floor.

References

EISENHART, C., HASTAY, M. & WALLIS, W. (Eds) (1947) *Techniques of Statistical Analysis for Scientific and Industrial Research and Production and Management Engineering* (New York, McGraw-Hill).

FUCHS, C. & KENETT, R.S. (1987) Multivariate Tolerance Regions and *F*-tests, *Journal of Quality Technology*, 19, pp. 122–131.

HALD, A. (1962) *Statistical Theory with Engineering Applications* (New York, John Wiley).

JACKSON, J. (1985) Multivariate quality control, *Communications in Statistics Part A—Theory and Methods*, 14, pp. 2657–2688.

JOHN, S. (1962) A tolerance region for multivariate normal distributions, *Sankhya, Series A*, pp. 363–368.

JOHNSTONE, D. (1987) Tests of significance in theory and practice (with discussions of G. Bernard and D. Lindley), *The Statistician*, 35, pp. 491–504.

KENETT, R. & HALEVY, A. (1984) Some statistical aspects of quality conformance inspection in military specifications documents, *Proceedings of 5th International Conference of the Israeli Society for Quality Assurance*, 6.2.1, pp. 1–10, Tel-Aviv.

LEURGANS, G. (1978) Evaluating laboratory measurement techniques, in: Miller, Jr, R.G., Efron, B., Brown, Jr, B.W. (Eds) *Biostatistics Casebook* (New York, John Wiley).