# ROBUST DESIGN USING COMPUTER EXPERIMENTS

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## 1 INTRODUCTION

Competitive products must meet strict standards for quality and reliability. Robust design experiments are an important quality engineering tool for developing low-cost, yet high quality products and processes. The name "robust design" derives from the idea of making products insensitive, or "robust", to the effects of natural variations, thereby reducing variation. Robust design was pioneered by Genichi Taguchi (Taguchi 1986) in Japan and has been embraced by many engineers around the world in the last 20 years. See Steinberg (1996) for detail on robust design and relevant chapters in Kenett and Zacks (1998) for background on design of experiments, robust design and reliability analysis. In this work we compare several different strategies for robust design when the experiment is carried out via a computer simulator. In these experiments, many different levels can be assigned to each factor and (ignoring numerical issues) running the simulator at the same set of inputs gives the same output. See Sacks et al. (1989), Bates et al. (1996) and Romano and Vicario (2002) for background and insight on the statistical aspects of computer experiments. The next section provides details of the simulation model that will be used throughout the paper. The following sections include a description of the methods and details of the results achieved with each method. We then compare the methods for a particular robust design task.

## 2 THE PISTON SIMULATOR

We analyze a simulator of a piston developed by Kenett and Zacks (1998). The response variable is cycle time of a complete revolution of the piston's shaft. The piston's performance can be regulated by changing seven control factors: A) Piston weight, B) Piston surface area, C) Initial gas volume, D) Spring coefficient, E) Atmospheric pressure, F) Ambient temperature and G) Filling gas temperature. Each of these factors is constrained to lie within a stated range and the ability to set the nominal level of each factor has a known degree of precision (see Kenett and Zacks, 1998). Randomness in cycle times is induced by normally distributed variations of the seven control factors about their nominal values.

## 3 ROBUSTNESS STRATEGIES

In this section we briefly describe several alternative methods that have been proposed for achieving robust design. We will denote the $k$ inputs to the simulator by $X_1, ..., X_k$ and will focus on a single output $Y$. Some of the inputs will be controllable factors (e.g. the thickness of an auto bumper or the material from which it is made) whereas other factors may be uncontrollable (e.g. the angle of impact with a crash barrier). The goal of the robustness strategies is to find nominal settings of the controllable input factors for which a desired output distribution is obtained. The output goal is often to achieve a given mean value with minimal variance. Taguchi's strategy for *robust design experimentation* is to include both design (controllable) factors and any noise factors that can be set to particular levels. Noise factors can represent either uncontrollable factors or tolerances of controllable factors about their nominal levels. Separate orthogonal arrays for the design factors and noise factors are combined to form a crossed-array design. The data are analyzed by computing signal-to-noise (SN) ratios that summarize the results at each control factor setting. Data from a crossed-array experiment can also be analyzed by the *response model* approach, which uses main effects for the noise factors and interactions between design factors and noise factors to study variation (Shoemaker, Tsui and Wu 1991, Sacks and Welch 1991, Steinberg and Bursztyn 1994). The *dual response surface approach* (Vining and Myers 1990) provides a direct way to assess jointly how the mean value and dispersion of $Y$ depend on the design factors. An experimental plan is prepared for the design factors only. Replicate measurements are generated from the simulator at each design point and are summarized by their average and a measure of spread, such as the standard deviation (SD), or its logarithm. Vining and Myers (1990) recommended a crossed-array design to obtain the replicates, but noted that they could also be sampled at random from distributions that are specified for the noise factors. Finally statistical models are built to relate the average and the measure of spread to the design factors. The *stochastic emulator approach* is based on the idea of replacing the original simulator, which may be expensive to evaluate, by a fast empirical surrogate, known as an emulator. The first step in this approach is to model the dependence of the output $Y$ on the full set of input values, whether or not they are controllable, to produce an emulator $\hat{Y}(X_1, \ldots X_k)$. Subsequent investigation proceeds by using the emulator to generate output rather than the simulator. The output distribution for any set of factor values can be approximated by sampling values for the noise factors and evaluating the emulator rather than the actual simulator. The final step involves choosing a feature of the output distribution, called the *stochastic response*, that represents the required robustness criteria. The stochastic response, as in the dual response surface approach, could be the SD or any other relevant feature of the output distribution. The rationale for the stochastic emulator approach is that complex simulators may take considerable time and computing resources to generate even a single output value. Thus the project sample size is determined by computing resources. The Stochastic Emulator approach has the potential advantage of devoting all of the computing resources to studying how the simulator output depends on the input factors. The first step is to run an experiment using all the input factors. Research to date has recommended “space filling” designs like Latin Hypercube Sampling (LHS) designs or lattice designs. See Sacks et al. (1989) and Bates et al. (1996) for more detail on designs. The emulator can be of any model type such as kriging estimators (Sacks et al. 1989), radial basis functions (Bates and Wynn 2000) or polynomial functions (Bates et. al. 2003).

## 4 COMPARISON OF ROBUSTNESS STRATEGIES ON THE PISTON

We applied the various robust design methods outlined above to the piston simulator. We purposely used designs with the same number of function calls for each method, in line with our comment that computing resources should dictate experiment size. The objective in all cases was to achieve a mean cycle time of 0.20 seconds while minimizing SD. Table 1 presents recommended factor settings derived from each of the analysis methods along with the results of 1000 actual simulator runs at those conditions. We begin with a brief explanation of how the recommended settings were obtained. The Taguchi analysis indicated that setting the surface area (B) to a high value is beneficial for both the SD and mean responses and setting the ambient temperature (F) to its highest value reduces variation and has almost no effect on the mean. Increasing the initial gas volume (C) to its highest value also reduces variation but moves the mean well above 0.2. A small amount of trial and error shows that setting C to 0.0044 achieves an estimated mean of about 0.2. The recommendations from the response model analysis for reducing variation were to set factors B, C and D, which have the strongest effects on cycle time, to high values. However, to achieve the desired average cycle time of 0.2 seconds, it was necessary to adopt a lower nominal value for C. The response model analysis did not find any important effect for F, which was set arbitrarily to its mid-range. The dual response surface designs, with replicates sampled at random from the noise distributions, were less successful than the cross-product design of the first two analyses in identifying factor settings that provide a mean value of 0.20 with a small SD. The analysis of the $2^{7-3}$ design favored high settings for B, which reduce both the mean value and the

SD. As no other effects were found on the SD, the remaining factors could be set to adjust the mean to the target value. Table 1 shows three possible solutions, each based on using factors C and D, to adjust the mean. All of the proposed solutions have means that are more than one SD above the target value. In addition, the SD's are about 25% larger than those achieved in the other analyses. For the 32-run LHS design, we used kriging models for the mean and SD as inputs to an optimizer, with the goal of minimizing the SD subject to maintaining the mean at 0.20. The resulting solution is shown in Table 1. Although these factor settings had excellent predicted performance (based on the kriging models), their actual performance is not good, with the mean more than 2.5 SDs off target. The stochastic emulator used a 64-point LHS design to build the emulator from simulator results. Monte Carlo analysis of the emulator was then used to estimate the response distribution at input factor values, using a 128 point LHS design to cover the factor space with 200 point samples from the noise distributions at each LHS point. New stochastic emulators were then built to predict the mean and SD. A constrained optimization was then performed, minimizing the stochastic emulator of the SD, while requiring that the emulator of the mean satisfy the constraint of equality to 0.2. The recommended factor settings and the results from actually running the simulator are shown in Table 1.

| | A | B | C | D | E | F | G | Mean | Std. Dev. |
|---|---|---|---|---|---|---|---|---|---|
| Taguchi | 30.3 | 0.017 | 0.00440 | 4850 | 100,000 | 295.6 | 350.0 | 0.204 | 0.0106 |
| Response Model | 30.3 | 0.017 | 0.00440 | 4850 | 100,000 | 293.0 | 350.0 | 0.204 | 0.0110 |
| Dual Response $2^{7-3}$ | 45.0 | 0.017 | 0.00275 | 3426 | 100,000 | 293.0 | 350.0 | 0.218 | 0.0137 |
| | 45.0 | 0.017 | 0.00488 | 4850 | 100,000 | 293.0 | 350.0 | 0.263 | 0.0137 |
| | 45.0 | 0.017 | 0.00382 | 4138 | 100,000 | 293.0 | 350.0 | 0.242 | 0.0132 |
| Dual Reponse LHS | 34.5 | 0.014 | 0.00346 | 4245 | 109,700 | 294.6 | 355.9 | 0.230 | 0.0154 |
| Stochastic Emulator I | 30.3 | 0.017 | 0.00359 | 3924 | 101,610 | 294.5 | 340.4 | 0.199 | 0.0107 |

**Table 1: Results for all methods for the problem of minimizing cycle time standard deviation about a target mean value of 0.2 seconds.**

For the piston example, the Taguchi method, the response model analysis and the stochastic emulator all provided better solutions than the dual response method. In particular, they did a much better job of keeping the mean cycle time on target. The dual response surface methods estimate the mean value at a given design point by taking a small sample of results from the noise distributions for the input factors. Our analysis suggests that small random samples do not provide sufficiently precise estimates of the mean cycle time. The stochastic emulator has the advantage of modeling the simulator directly from function evaluations and produced an excellent robust design solution for our trial problem. The results highlight the need to consider how best to allocate resources when conducting computer experiments. Our results with the stochastic emulator approach indicate that this may be an efficient method for reducing the number of simulations in a robust design study.

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